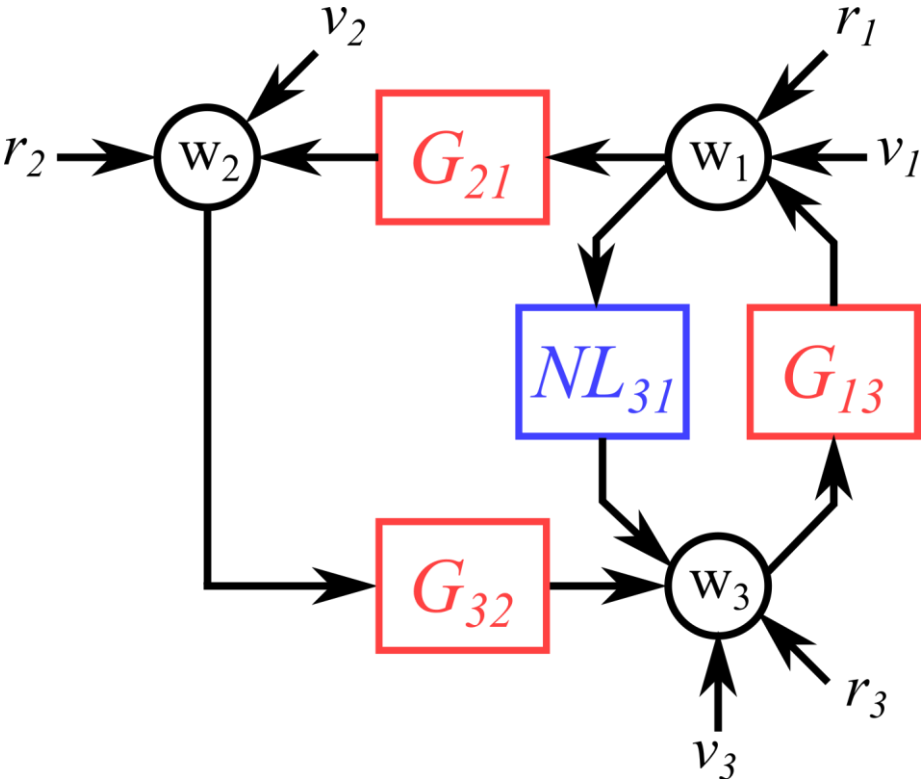


# Detecting and Quantifying Nonlinearity in a Dynamic Network through the Best Linear Approximation

M. Schoukens, P.M.J. Van den Hof



# "small data" system identification

	# of samples →		
problem	ill-posed	ill-conditioned	well-cond.
cost fun.	—	local minima	→ convex
solution	non-unique	sensitive	robust
theoretical results	—	few results	asymptotic analysis

proposed method:

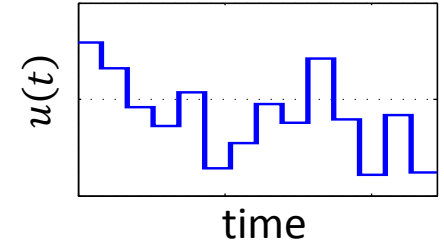
1.  $(u, y) \mapsto h$  — impulse response estimation  
using prior knowledge
2.  $h \mapsto \text{model}$  — realization

# Computing the D-optimal Zero Order Hold Input for Wiener Models with fixed Power Nonlinearity

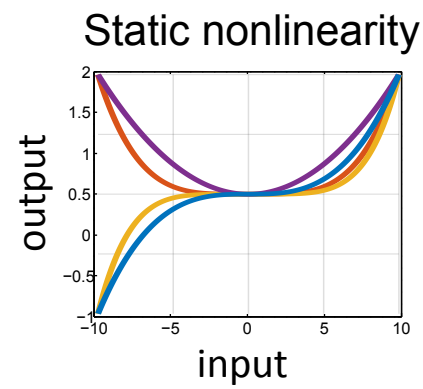
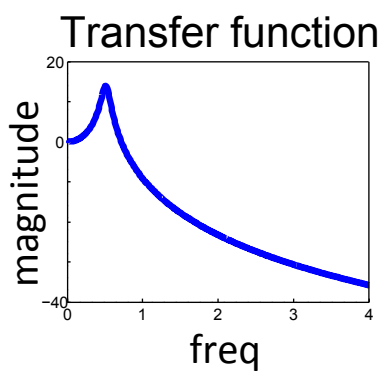
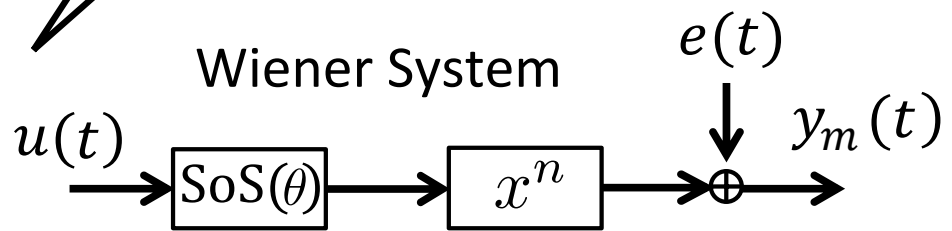
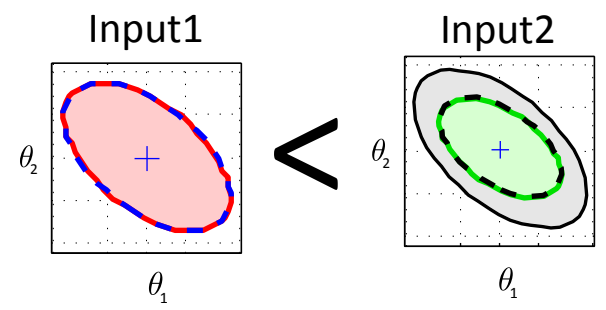
What is the most informative input to excite this system?

What is the input class?

ZOH signals



What is D-optimal?



# Decoupling nonlinear black-box models using tensor methods

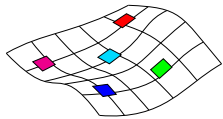
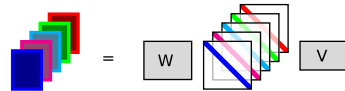
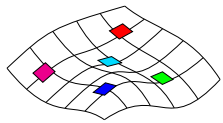
Philippe Dreesen

Mariya Ishteva

Johan Schoukens

Vrije Universiteit Brussel (VUB)

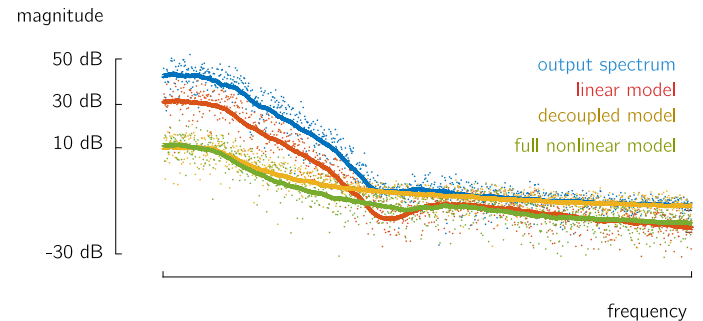
$$\begin{bmatrix} \mathbf{x}^+ \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} + \boxed{\mathbf{f}(\mathbf{x}, \mathbf{u})}$$



$$\begin{bmatrix} \mathbf{x}^+ \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} + \begin{matrix} \boxed{W} & \begin{matrix} g_1(x_1) \\ g_r(x_r) \end{matrix} & \boxed{V^T} \end{matrix}$$

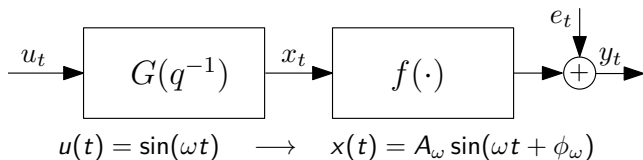
Decoupling nonlinear MIMO functions

- model interpretability
- reducing number of parameters



# Two-experiment approach to Wiener system identification

G. Bottegal (TU/e), R. Castro-Garcia (KUL), J.A.K. Suykens (KUL)



Estimation of  $f(\cdot)$  = regression problem + additional parameter  $\phi_\omega$

- 1 Parametric approach for general basis functions
- 2 Least-squares approach for polynomial nonlinearity
- 3 Nonparametric approach

# Gray-Box Identification using Difference-of-Convex Programming

Work of Chengpu Yu (Beijing Institute of Technology)

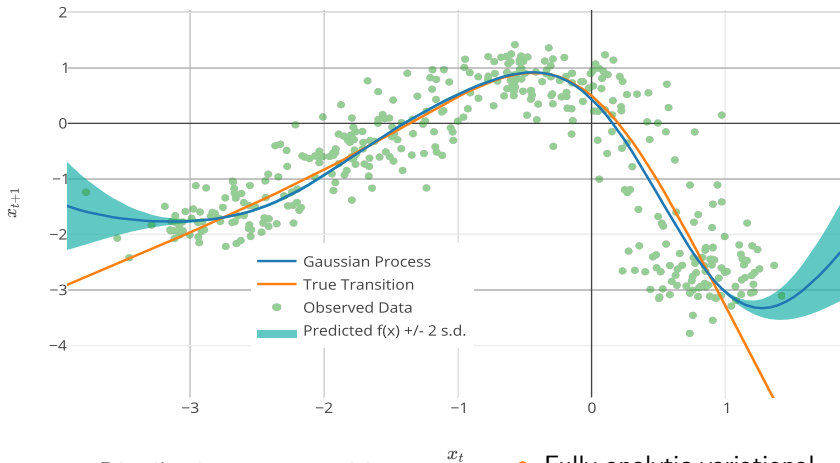
Together with Prof. Lennart Ljung and Michel Verhaegen

September 25, 2017 at the ERNSI meeting in Lyon



# Learning Non-linear Dynamics

A. D. Ialongo, M. van der Wilk, C. E. Rasmussen



- Distribution over transition functions
- Infer system dimensionality
- Impute missing information
- Fully analytic variational approximation
  - ▶ No need for sampling, inference by optimisation



# Complementary and extended Kalman filters for orientation estimation

Manon Kok<sup>1</sup> and Thomas B. Schön<sup>2</sup>

1: Department of Engineering, University of Cambridge, UK  
2: Department of Information Technology, Uppsala University, Sweden



UPPSALA  
UNIVERSITET

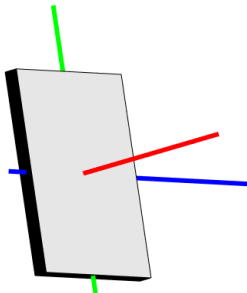


We study the relationship between

- ▶ complementary filters
- ▶ and extended Kalman filters

for orientation estimation using inertial and magnetometer measurements.

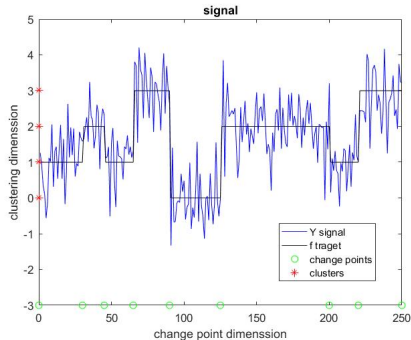
**Contribution:** A complementary filter highlighting similarities and differences between these two filters.





## Model selection for change point detection and clustering

**Problem:** estimate the mean of a signal, assuming it is piece-wise constant and can take only a small number of levels.

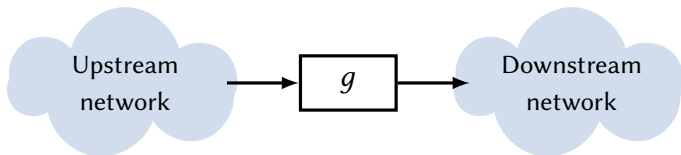


In the poster:

- ▶ Bayesian selection framework for this problem
- ▶ Model selection criteria for the estimator
- ▶ Oracle inequality for the estimator
- ▶ An adaptive upper bound of the risk
- ▶ And more...

## Distributed nonparametric identification of acyclic dynamic networks

Riccardo Sven Risuleo and Håkan Hjalmarsson



*Can we estimate  $g$  using local information only?*



# Learning an optimization solver for a class of inverse problems

Jonas Adler, Johan Karlsson, Axel Ringh, and Ozan Öktem

- Given family of cost functions  $H_b(x) = \|Ax - b\|_2^2 + \|\Delta x\|_1$  we train an optimization solver to perform as good as possible for fixed number of iterations.

- We extend this by parameterizing a family of algorithms and learn parameters for minimizing a function given a fixed number of iterations.

Method	$H_b(x_{30})$
Sidky et al [7]	9.322
Const. params.	9.253
Free params.	9.234
$N = M = 2$	9.220
$N = M = 3$	<b>9.218</b>



# Another particle-filter approach to maximum likelihood estimation

## State-space model

$$x_t \mid x_{t-1} \sim f_\theta(x_t \mid x_{t-1})$$

$$y_t \mid x_t \sim g_\theta(y_t \mid x_t)$$

## Estimation

$$\hat{\theta} = \arg \max_{\theta} p(y_{1:T} \mid \theta)$$

## Using particle filters

- ☺ Unbiased estimates of  $p(y_{1:T} \mid \theta)$
- ☹ Stochastic estimates of  $p(y_{1:T} \mid \theta)$

→ Showstopper for conventional optimization

## Question

If we run the particle filter with some  $\theta_{k-1}$ , can we re-use these particles to also estimate  $p(y_{1:T} \mid \theta')$ ? ( $\theta'$  in the neighborhood of  $\theta_{k-1}$ )

## Spoiler

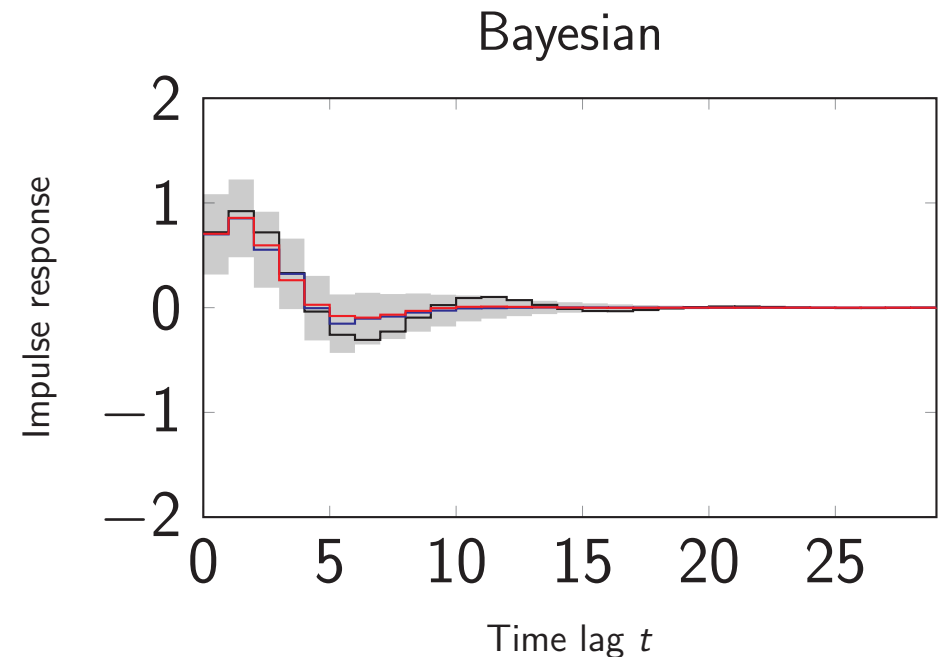
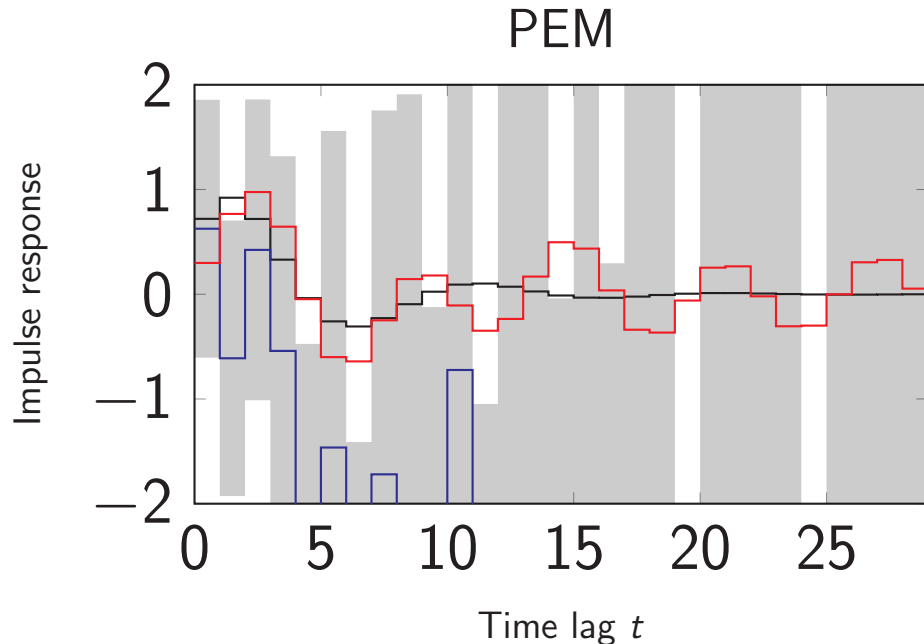
Yes, and it allows for conventional optimization tools to be applied.

# Decision-Theoretic Approach to System Identification

Johan Wågberg, Dave Zachariah, Thomas B. Schön

Department of Information Technology, Uppsala University

- ▶ Classical approach: Parametric prediction error methods (PEM).
- ▶ Modern approach: Regularized methods for impulse response estimation.
- ▶ By viewing identification as a decision we develop a decision-theoretic framework that bridges the gap.
- ▶ Output error model class serves as an illustration.



# Model Reduction for Linear Bayesian SysId

G. Prando



A. Chiuso

Non-parametric Bayesian SysId estimates high-order FIR models



**Model Reduction** is needed  
for control and filtering applications

We compare:

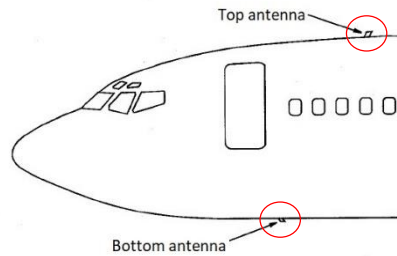
- 2 reduction algorithms
- several criteria for choosing the order of the reduced model

# Grey-box identification for active vibration control of a flexible structure with piezo patches

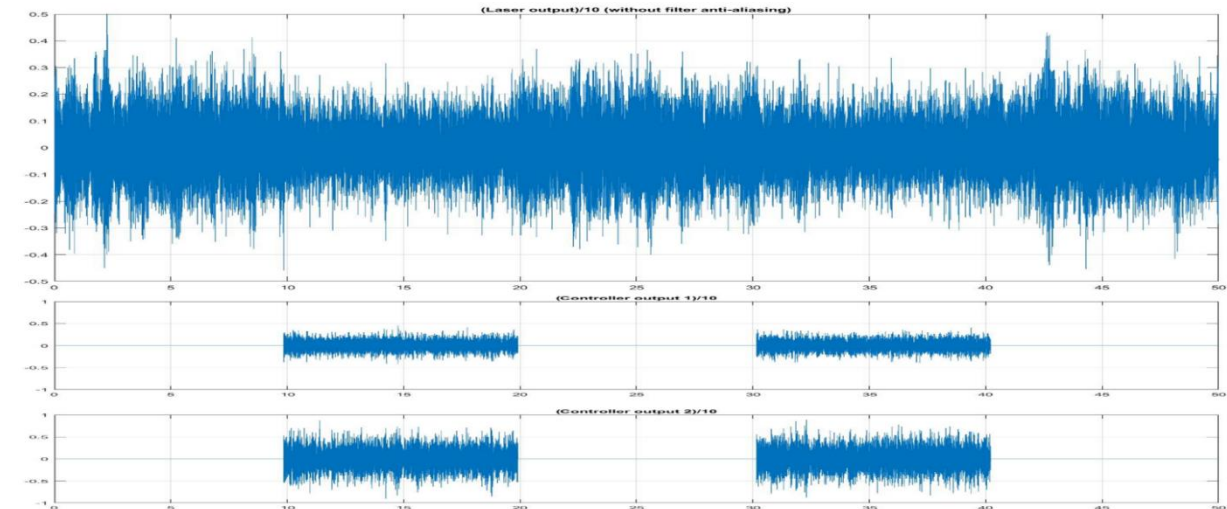
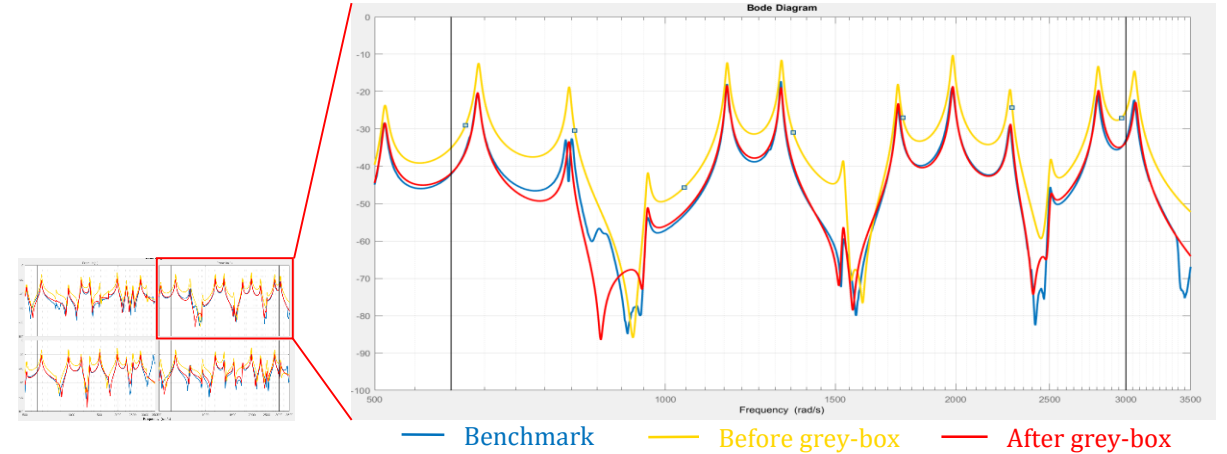
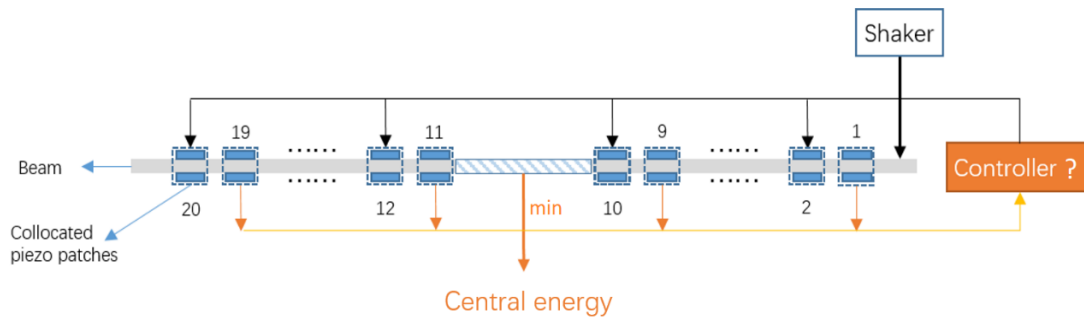
P. WANG (ECL), C. Wang (ECL), A. Kornienko (ECL), G. Scorletti (ECL), X. Bombois (CNRS), Manuel Collet (ECL)

## Non-typical problem:

1. the vibration energy must be particularly rejected in a specific location
2. piezo-transducers cannot be placed at this location

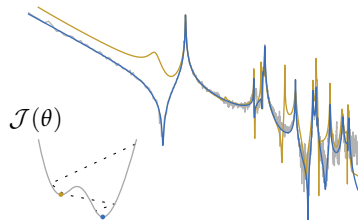
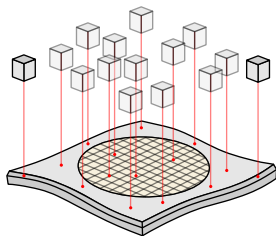
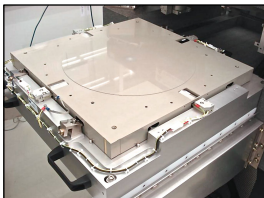


- MIMO feedback controller
- Minimize central energy
- Guaranteed performance
- High quality model: Identification



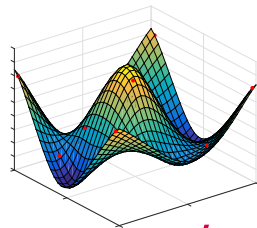
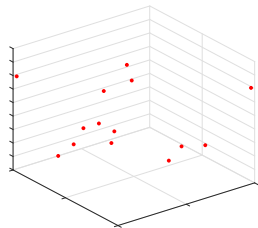
# Parametrizing Mechanical Systems using Matrix Fraction Descriptions

ERNSI 2017



Question: "When is  $P(\theta, \xi) = D^{-1}(\theta, \xi)N(\theta, \xi)$  equivalent to  $\mathcal{L}[\xi^2 I + D_m \xi + \Omega^2]^{-1} \mathcal{R}$  ?"

$$\left\{ \begin{array}{cc|c} 0 & I & 0 \\ \Omega^2 & D_m & \mathcal{R} \\ \hline \mathcal{L}(\rho_1) & 0 & 0 \\ \vdots & \vdots & \vdots \\ \mathcal{L}(\rho_{n_\rho}) & 0 & 0 \end{array} \right.$$

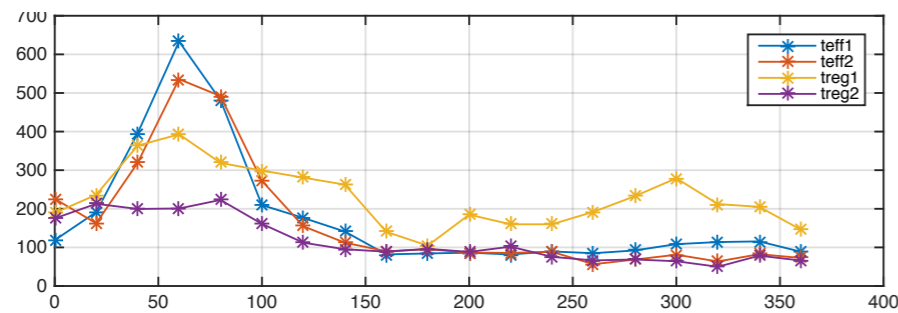




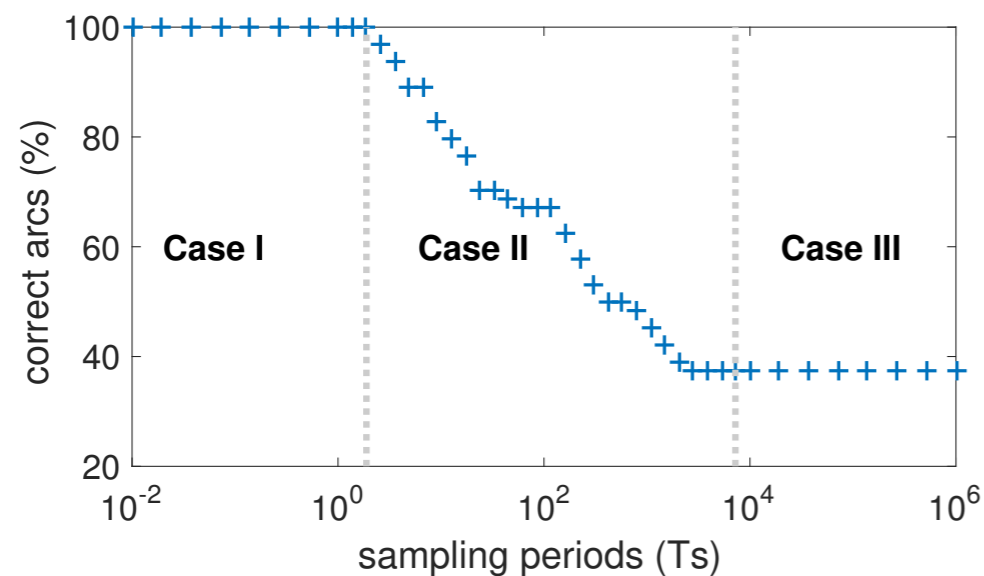
Zuogong Yue, Jorge Goncalves

Luxembourg Centre for Systems Biomedicine (LCSB), University of Luxembourg

## Examples of Time series



## Failure of Discrete-time Methods



## LTI Network Model

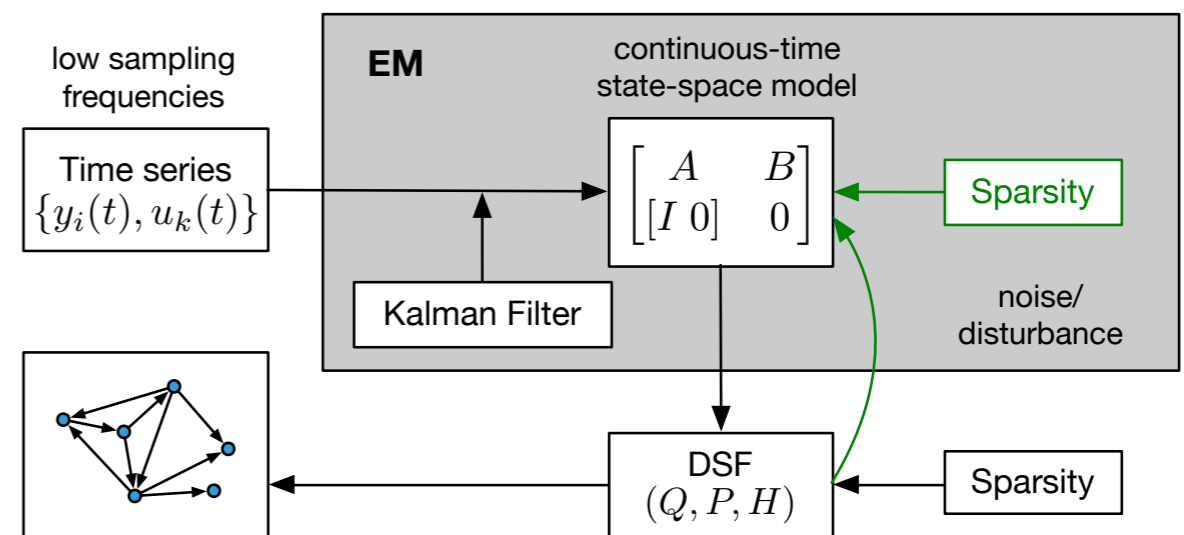
(Dynamical Structure Functions)

Causal/dynamic network is inferred by identifying

$$y(t) = Q(q)y(t) + P(q)u(t) + H(q)e(t)$$

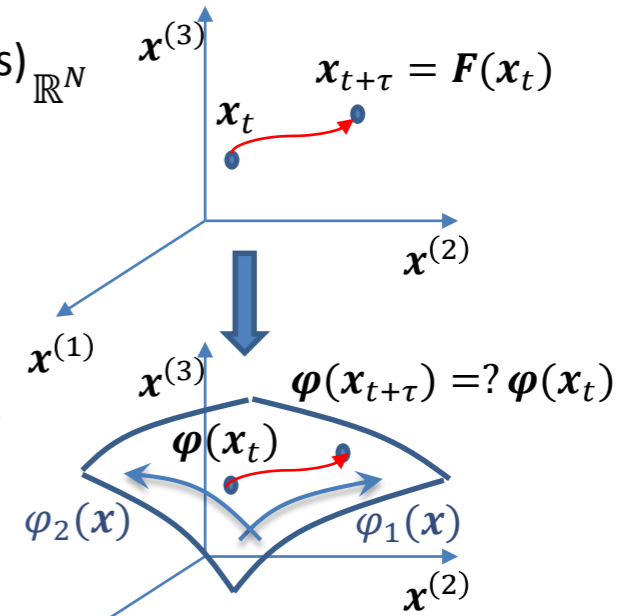
where  $Q, P, H$  are matrices of transfer functions,  $q$  is the forward-shift/differential operator.

## Framework



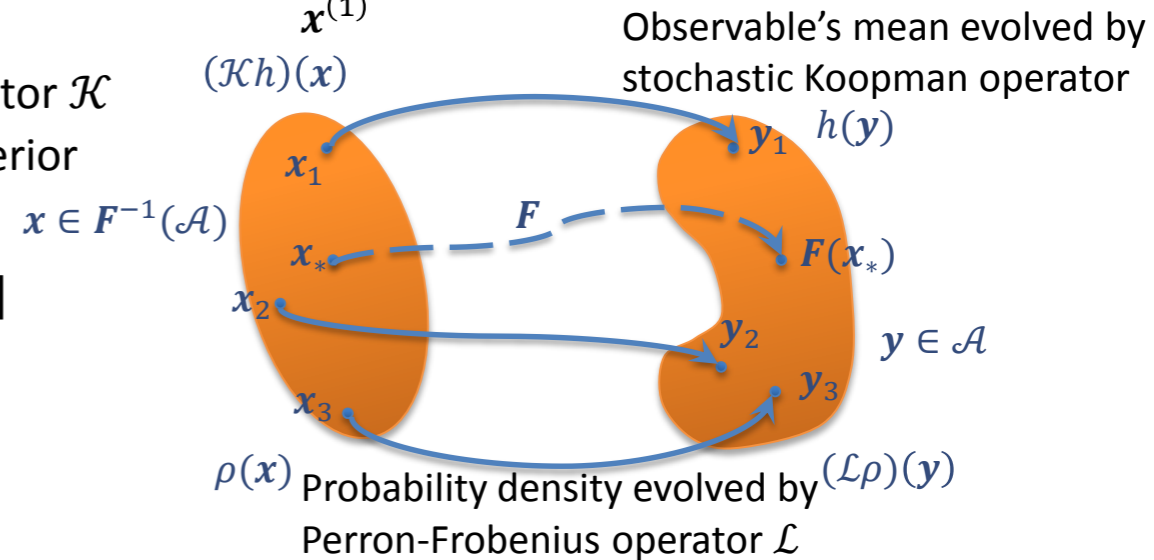
## Main ideas:

- High-dimensional time series (with exogenous variables)  $\Leftrightarrow$  dynamical system (with inputs)  $\mathbb{R}^N$
- State variables  $\mathbf{x}_{t+\tau} = \mathbf{F}(\mathbf{x}_t)$ : high-dimensional extrinsic measurements/outputs of underlying lower dimensional true state's dynamics  $\mathbf{z}_{t+\tau} = \widehat{\mathbf{F}}(\mathbf{z}_t)$
- For prediction of outputs  $\mathbf{x}_{t+\tau}$ , learning  $\mathbf{F}$  without identifying  $\widehat{\mathbf{F}}(\mathbf{z}_t)$ : not optimal and computationally heavy.
- Want some intrinsic feature map  $\{\varphi_i(\mathbf{x}_t)\}$  (doesn't matter if  $\varphi(\mathbf{x}_t)$  and  $\mathbf{z}_t$  are same for prediction purpose) to embed  $\mathbf{x}_t$  to intrinsic manifold: learn *both* geometry *and* dynamics for *simultaneous* dimension reduction and prediction.
- These  $\{\varphi_i(\mathbf{x}_t)\}$  are eigenfunctions of Koopman operator  $\mathcal{K}$ ; transform  $\varphi(\mathbf{x}_{t+\tau})$  back to  $\mathbf{x}_{t+\tau}$  using Koopman modes.



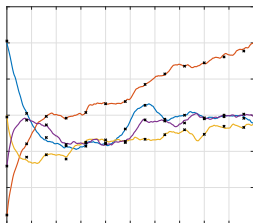
## Approximating Koopman operator in RKHS

- Choose RKHS as the feature/function space to approximate the linear operator  $\mathcal{K}$
- RKHS  $\Leftrightarrow$  Gaussian Processes Regression: point evaluation  $\langle k_{x_*} | h \rangle_{\mathcal{H}_k} \Leftrightarrow$  posterior mean (statistical interpretation)
- Given  $\mathbf{x}_*$  and training data,  $h(\mathbf{F}(\mathbf{x}_*)) = \mathcal{K}h(\mathbf{x}_*) = \langle k_{x_*} | \mathcal{K} | h \rangle_{\mathcal{H}_k} = \mathbb{E}[h(\mathbf{y})]$
- $\mathcal{K}$  in RKHS  $\Leftrightarrow$  stochastic Koopman operator and Perron-Frobenius operator

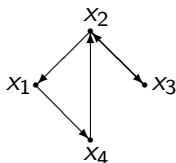


## Also includes:

- Algorithm to approximate  $\mathcal{K}$ : kernel-based EDMD
- Generalizing Koopman operator to system with inputs
- Numerical examples and prediction performance
- Summary of advantages
- Future outlooks



$$\Rightarrow \frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_2, x_3, x_4) \\ f_3(x_2, x_3) \\ f_4(x_1, x_4) \end{bmatrix} + u$$



- We model  $f_i$ 's as Gaussian processes with covariance functions  $k_i(t, s) = \gamma_i \exp\left(-\sum_{j=1}^n \beta_{i,j}(t_j - s_j)^2\right)$ .
- Estimate hyperparameters  $\beta_{i,j} \geq 0$  from the data.
- If  $\beta_{i,j} > 0$ , it means  $x_j$  regulates  $x_i$ .



# Performance Analysis for Stochastic Wiener System Identification: A Simple Yet Complicated Example

**Bo Wahlberg and Lennart Ljung**

- ▶ **Problem:** Identification of a stochastic linear dynamical system with a *non-linear measurement sensors* (Stochastic Wiener System)
- ▶ **Question:** How does the nonlinear characteristic of the sensor affect the accuracy of the estimated model?
- ▶ **Answer:** It can improve or deteriorate the accuracy compared to a linear sensor with the same gain!
- ▶ **How:** We will use Gaussian approximations and the corresponding Fisher Information Matrix for the performance analysis of a *simple yet complicated example*.