Local LTI Model Coherence for LPV Interpolation

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LPV systems identification

Local approach vs global approach

- Pros……
- Cons……

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LPV systems identification

Local approach vs global approach

- Pros……
- Cons……

But no time for those comments ☺

Let's focus on the local approach…

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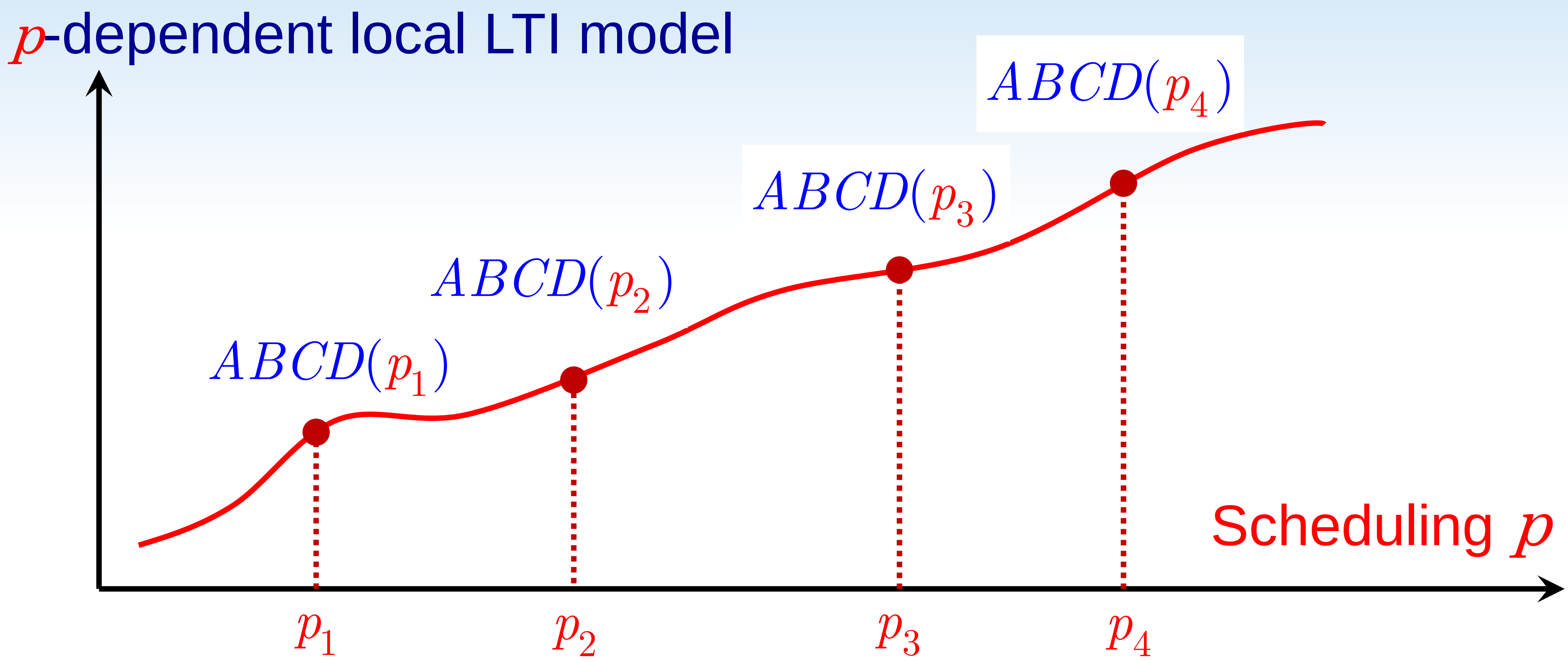
Local approach to LPV system identification

$$\begin{aligned} x_{k+1} &= A(p)x_k + B(p)u_k + w_k \\ y_k &= C(p)x_k + D(p)u_k + v_k \end{aligned}$$

Locally estimated state-space models



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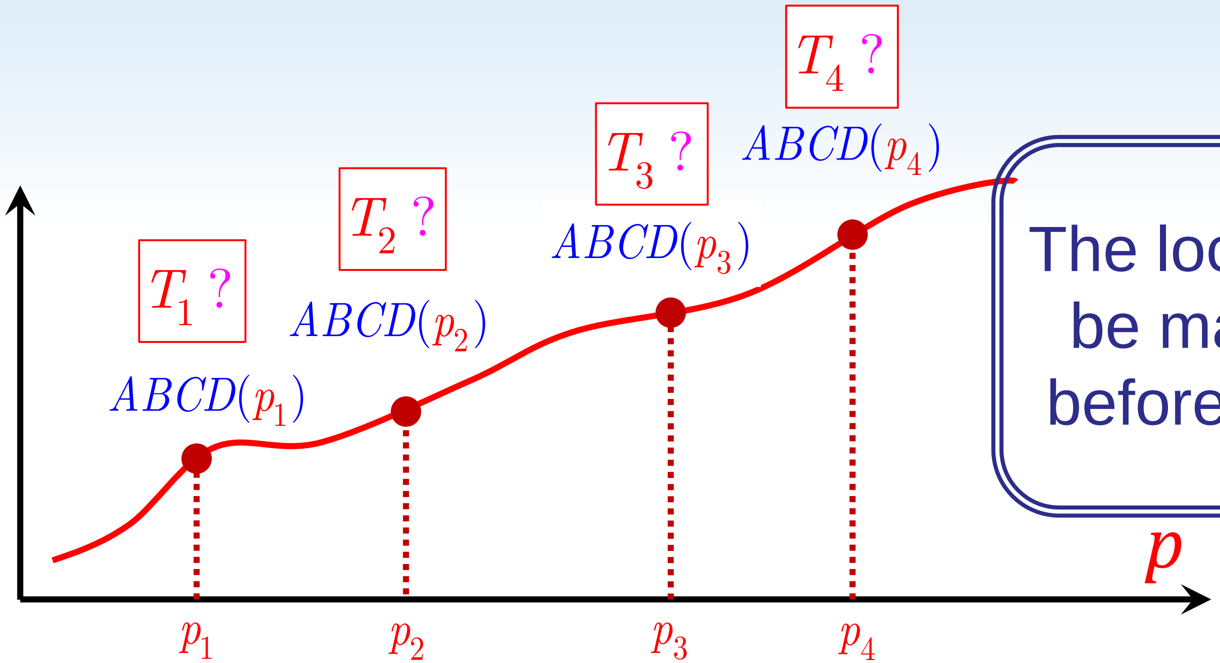
Local approach to LPV system identification

Only state-space models are considered in this presentation.

$$
\begin{aligned}
x_{k+1} &= A(p)x_k + B(p)u_k + w_k \\
y_k &= C(p)x_k + D(p)u_k + v_k
\end{aligned}
$$

Local LTI model interpolation

The model parameters $ABCD(p_i)$ are estimated from I/O data **up to an arbitrary similarity transformation** T_i.



The local models must be made “coherent” before **interpolation.**

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Local LTI model interpolation

It is widely acknowledged that state-space models must be made "coherent" before their interpolation.

But there is no consensus on how.

Local LTI model interpolation

It is widely acknowledged that state-space models must be made "coherent" before their interpolation.

But there is no consensus on how.

More troublesomely, what does mean a "coherent" set of local models?

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Local LTI model interpolation

This presentation is about the "**coherence**" of local LTI models for the purpose of their interpolation.

Model interpolation $\neq$ Output interpolation

Local LTI model interpolation

This presentation is about the "**coherence**" of local LTI models for the purpose of their interpolation.

An interesting, but controversial topic……

"Coherent" local models for interpolation

Local models are **usually** transformed into some **particular (canonical) form** before their interpolation:

- **The controllable canonical form**
  [Steinbuch et al., 2003]
- **The balanced form**
  [Lovera and Mercere, 2007]
- **The modal form**
  [Yung, 2002]
- **A zero-pole decomposition-based form**
  [De Caigny et al., 2009, 2011]
- **An observability matrix-based form**
  [De Caigny et al., 2014]

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"Coherent" local models for interpolation

In general, these "coherent" forms are **not compatible** with each other.

- The controllable canonical form
  [Steinbuch et al., 2003]
- The balanced form
  [Lovera and Mercere, 2007]
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Definitions of "Coherent" local models?

Local model "**coherence**" is **rarely defined** in the literature.

An existing definition. Regarding an LPV system

$$\begin{aligned} x_{k+1} &= A(p)x_k + B(p)u_k + w_k \\ y_k &= C(p)x_k + D(p)u_k + v_k \end{aligned}$$

a set of LTI models $\{(A_i, B_i, C_i, D_i), i = 1, \ldots, m\}$ are coherent if there exists **some** matrix-valued **function** $T(p)$ such that

$$A_i = T(p_i)A(p_i)T^{-1}(p_i) \quad \text{for} \quad i = 1, 2, \ldots, m$$

and similarly $B_i = \cdots, C_i = \cdots, D_i = \cdots$

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Comments on this definition

LPV

$$x_{k+1} = A(p)x_k + B(p)u_k + w_k$$
$$y_k = C(p)x_k + D(p)u_k + v_k$$

$$p \in \{p_1, p_2, \ldots, p_m\}$$

LTI

$$\{(A_i, B_i, C_i, D_i), i = 1, 2, \ldots, m\}$$

Indexed by i

- $\mathrm{LPV}(p_i) \xrightarrow{T_i} (A_i, B_i, C_i, D_i)$
- There exists **some** $T(p)$ such that $T_i = T(p_i)$
- $T(p)$ and $T^{-1}(p)$ are continuous and bounded

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Is this definition relevant?

Randomly generate square matrices of size $n = \dim(x)$, keep only those whose determinants are positive, and use them as T_i

$$\mathrm{LPV}(p_i) \text{ — random } T_i \longrightarrow (A_i, B_i, C_i, D_i)$$

Is this definition relevant?

Then, according to the previous definition, these randomly transformed local models are coherent!

Interpolation $\rightarrow T(p)$

$\mathrm{LPV}(p_i)$ — random T_i $\rightarrow$ (A_i, B_i, C_i, D_i)

Too much freedom is left to T_i :

There exists **some** $T(p)$ such that $T_i = T(p_i)$.

Is this definition relevant?

There exists **some** $T(p)$ such that $T_i = T(p_i)$.

Moreover, the p-dependent $T(p)$ leads, in general, to dynamic parameter dependence.

Discrete time

$$\mathrm{LPV}(p_k) \xrightarrow{\quad T(p) \quad} \mathrm{LPV}(p_k, p_{k+1})$$

Static p-dependent — Dynamic p-dependent

Continuous time

$$\mathrm{LPV}(p(t)) \xrightarrow{\quad T(p) \quad} \mathrm{LPV}(p(t), \dot{p}(t))$$

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How should be a relevant coherence definition?

$\mathrm{LPV}(p)$ ⟷ "Close" I/O behavior ⟷ Interpolated coherent LTIs(p)

In particular, when p **evolves** within $\{p_1, p_2, \ldots, p_m\}$, no interpolation is necessary:

$\mathrm{LPV}(p)$ ⟷ **Same** I/O behavior ⟷ Coherent LTIs (p)

A **new** coherence definition proposal

$$\mathrm{LPV}(p) \text{ frozen at } p \in \{p_1, p_2, \ldots, p_m\}$$

Given a set of LTI models $\{(A_i, B_i, C_i, D_i), i = 1, 2, \ldots, m\}$,

if there exists a transformation matrix T **common to all** the m LTI models, such that

$$\mathrm{LPV}(p_i) \xrightarrow{\;T\;} (A_i, B_i, C_i, D_i)$$

then the set of LTI models are coherent.

Difference between the two definitions

New definition

$\mathrm{LPV}(p_i) \xrightarrow{T} (A_i, B_i, C_i, D_i)$

Previous definition

Indexed by i

$\mathrm{LPV}(p_i) \xrightarrow{T_i} (A_i, B_i, C_i, D_i)$

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Property of the proposed definition

When p **evolves** within $\{p_1, p_2, \ldots, p_m\}$,

This property is only a “necessary condition” for a relevant coherence definition.

Are there $T(p)$-based definition satisfying this property?

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p-dependent transformation?

In general, p-dependent transformations $T(p)$ lead to dynamic p-dependent LPV models.

Consequently, transformations like $A_i = T_i A(p_i) T_i^{-1}$ do not preserve I/O property, **in general**.

p-dependent transformation?

In general, p-dependent transformations $T(p)$ lead to dynamic p-dependent LPV models.

Consequently, transformations like $A_i = T_i A(p_i) T_i^{-1}$ do not preserve I/O property, in general.

However, there do exist $T(p)$ leading to static p-dependent LPV models.

Should such transformations be included in a definition of coherent local models?

p-dependent transformation?

If two static affine p-dependent and state-minimum LPV systems have the same I/O property, then they are related by a p-independent linear transformation. [Petreczky and Mercère 2012]

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p-dependent transformation?

If two static affine p-dependent and state-minimum LPV systems have the same I/O property, then they are related by a p-independent linear transformation. [Petreczky and Mercère 2012]

Finite p-value set → affine p-dependence

For state-minimum models, no generalization to $T(p)$ of the coherence definition is possible.

p-dependent transformation?

If the matrix **sum**

Sum of positive (semi)definite matrices

$$\sum_{i=1}^{m} A(p_i)A^T(p_i) + B(p_i)B^T(p_i)$$

is non singular, then no generalization to $T(p)$ of the coherence definition is possible.

A simpler, but stronger, **yet reasonable**, condition: any of the $A(p_i)$ is non singular.

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p-dependent transformation?

⇒ In most situations, the p-independent **coherence definition** is the only possible one.

Next question

Now the definition of coherent local models is clarified, how can we find the linear transformations making locally estimated models coherent?

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Making local LTI models coherent?

If the locally estimated LTI models are structurally independent, then it is impossible to determine linear transformations making them coherent, solely based on these LTI models.

[Zhang and Ljung, Automatica, 2017].

Making local LTI models coherent?

This disappointing result means that such local LTI models do not contain sufficient information to make themselves coherent.

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Making local LTI models coherent?

This disappointing result means that such local LTI models do not contain sufficient information to make themselves coherent.

Nevertheless, to make local LTI models coherent, it is possible to use

- global structural assumptions, and/or
- I/O data under some excitation condition.

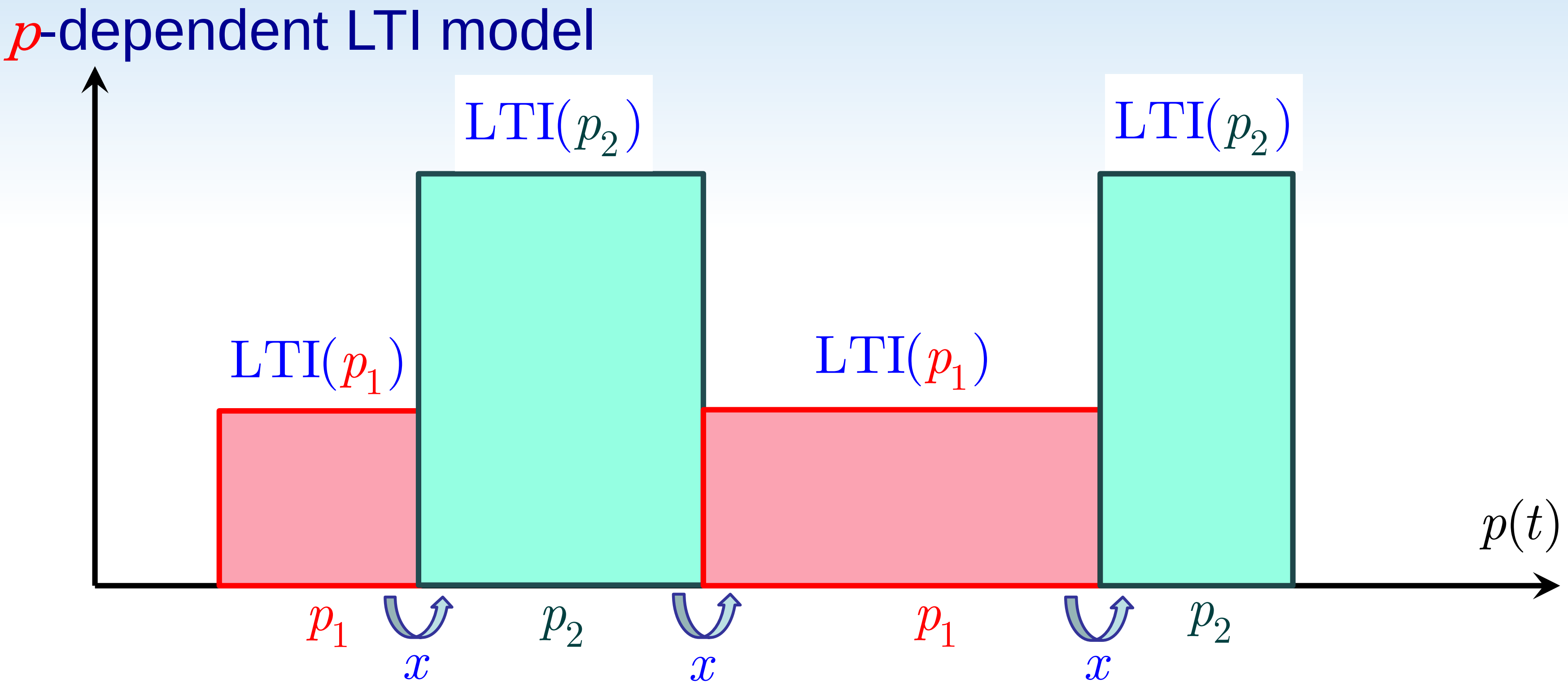
Make structural assumptions

The LPV interpolation methods based on **particular (canonical) forms** must be (**implicitly**) based on some global structural assumptions.

- The controllable canonical form
- The balanced form
- The modal form
- A zero-pole decomposition-based form
- An observability matrix-based form

Use I/O data sequence

Within a data sequence, the state "continuity" at p-transitions provides constraints on state bases of local LTI models.



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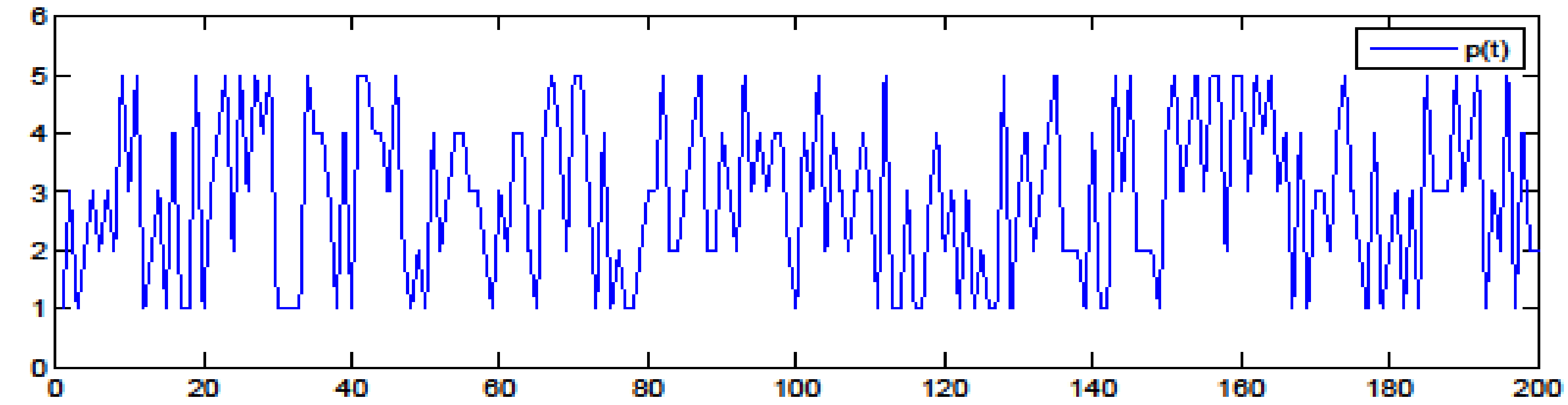
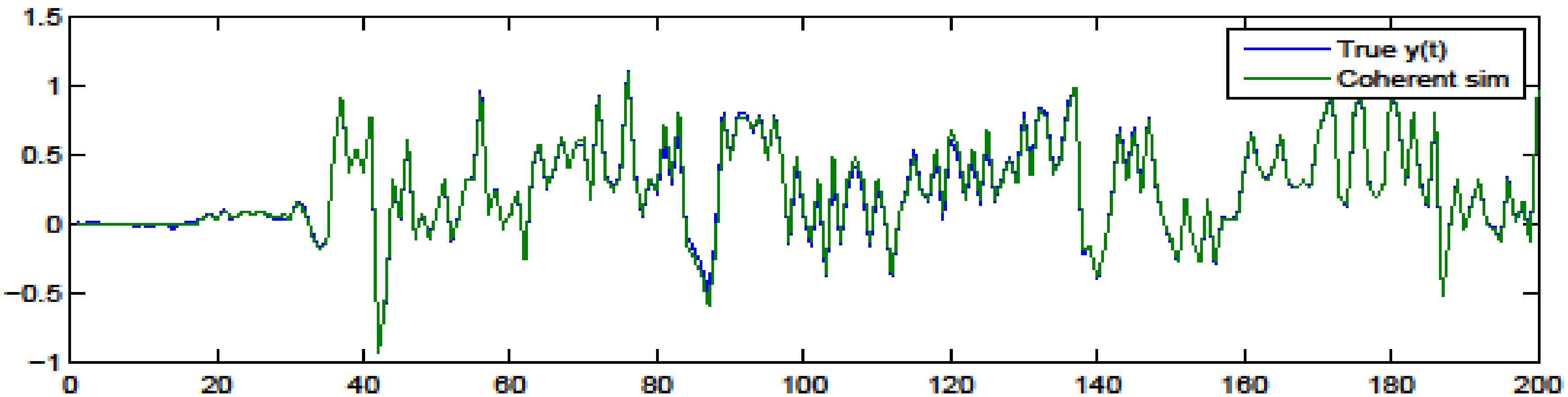
Use I/O data: numerical example

Simulation with coherent LTI models for **fast** p-transitions.





A few words about interpolation of I/O models

Interpolating local I/O models avoids the problem of coherence, but the resulting LPV model is not suitable for fast p-transitions.

In applications involving model-based simulation (*e.g.*, MPC), somehow a state-space form is necessary, to manage the state “continuity” at p-transitions.

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Summary

- ➢ Local model coherence: definition clarified.
- ➢ Structurally independent local models do not contain the information to make themselves coherent.
- ➢ Locally estimated LTI models can be made coherent based on
  - ✓ Global structural assumptions
  - ✓ I/O data sequences under some excitation condition.

Summary

- Local model coherence: definition clarified.
- Structurally independent local models do not contain the information to make themselves coherent.
- Locally estimated LTI models can be made coherent based on
  - ✓ Global structural assumptions
  - ✓ I/O data sequences under some excitation condition.

Is this still a local approach?

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